LECTURER4

Some obvious Limits

(1)If k is constant,then $\lim_{x\to+\infty} f(x)=k$ and $\lim_{x\to-\infty} f(x)=k$

(2) $\lim_{x\to\infty}\frac{1}{x}=0$, $\lim_{x\to+\infty}\frac{1}{x}=0$,and) $\lim_{x\to-\infty}\frac{1}{x}=0$

(3)) $\lim_{x\to0}\frac{1}{x}=\infty$, (3)) $\lim_{x\to0+}\frac{1}{x}=\infty$,and (3)) $\lim_{x\to0-}\frac{1}{x}=\infty$

Examples:

Find the following limits

(1) $\lim_{x\to\infty}\frac{x}{2x+3}=\lim_{x\to\infty}\frac{1}{2+\frac{3}{x}}=\frac{1}{2}$

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(2) $\lim_{x\to\infty}(\sqrt{x^2+1}-x=\lim_{x\to\infty}(\sqrt{x^2+1}-x.\frac{\sqrt{x^2+1}+x}{(\sqrt{\sqrt{x^2+1}+x}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x}}+1}=\frac{0}{2}=0$

LECTURER4

Theorems on limits(Calculation Technique)

1-Uniqueness of limit

If $\lim\limits_{x\to a} f(x) = L$ then L is unique

2-Limit of constant

If f(x)=c, where c is constant then $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} c = c$

3-Obvious limit

If f(x)=x then $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} x = a$

4-Limit of Som

If $f(x) = f_1(x) \pm f_1(x) \pm f3(x) \pm \ldots\ldots\ldots \pm f_n(x)$ and $\lim\limits_{x\to a} f_i(x) = L_i$,i=1,2,….,n,then

$$\lim_{x\to a} f(x) = \lim_{x\to a} f_1(x) \pm \lim_{x\to a} f_2(x) \pm \ldots \lim_{x\to a} f_n(x) = L_1 \pm L_2 \pm \ldots\ldots \pm L_N$$

5-Limit of product

If $f(x) = f_1(x).f_2(x).......f_n(x)$ and $\lim\limits_{x\to a} f_i(x) = L_i$, i=1,2,….,n,then $\lim\limits_{x\to a} f(x) =$ $\lim\limits_{x\to a} f_1(x).\lim\limits_{x\to a} f_2(x).\ldots\ldots\lim\limits_{x\to a} f_n(x). = L_1.L_2.......L_n$

6-Limit of Quotient

If $f(x)=\dfrac{g(x)}{h(x)}$ and $\lim\limits_{x\to a} g(x)=L_1$,and $\lim\limits_{x\to a} h(x)=L_2$, $L_2\neq 0$ then $\lim\limits_{x\to a} f(x)=$

$$\lim_{x\to a} f(x)=\frac{\lim\limits_{x\to a} g(x)}{\lim\limits_{x\to a} h(x)}=\frac{L_1}{L_2}$$

EXAMPLE 3 Using Properties of Limits

Use the observations $\lim_{x\to c} k=k$ and $\lim_{x\to c} x=c$, and the properties of limits to find the following limits.

(a) $\lim\limits_{x\to c}(x^3+4x^2-3)$ **(b)** $\lim\limits_{x\to c}\dfrac{x^4+x^2-1}{x^2+5}$

SOLUTION

(a) $\lim\limits_{x\to c}(x^3+4x^2-3)=\lim\limits_{x\to c} x^3+\lim\limits_{x\to c} 4x^2-\lim\limits_{x\to c} 3$ Sum and Difference Rules

$=c^3+4c^2-3$ Product and Constant Multiple Rules

(b) $\lim\limits_{x\to c}\dfrac{x^4+x^2-1}{x^2+5}=\dfrac{\lim\limits_{x\to c}(x^4+x^2-1)}{\lim\limits_{x\to c}(x^2+5)}$ Quotient Rule